

## About the model of mechanical dynamics on road motor vehicles with damping pillows installed

Về mô hình động lực học hệ máy móc trên phương tiện cơ giới đường bộ có lắp gối giảm chấn

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### Abstract

The issue of research on improving the working reliability of specialized machinery and equipment placed on road motor vehicles subject to vibration has been of interest and studied in many countries around the world. A number of recently published research results have demonstrated the research direction to reduce vibrations by isolating methods. Active control brings high efficiency and is suitable to achieve stable targets for equipment on this motor vehicle. This paper presents the results of building an oscillation analysis tool on a motor vehicle with an oscillating isolation pillow as a basis for calculating and designing electromagnetic damping pillows to reduce vibrations on this motor vehicle.

**Keywords:** Dynamic model; road motor vehicles; reducing vibration pillows.

### Tóm tắt

Vấn đề nghiên cứu nâng cao độ tin cậy làm việc của các máy móc, thiết bị chuyên dùng đặt trên phương tiện cơ giới đường bộ chịu tác động rung động đã được nhiều nước trên thế giới quan tâm và nghiên cứu. Một số kết quả nghiên cứu được công bố gần đây đã minh chứng hướng nghiên cứu giảm dao động bằng phương pháp cách ly, điều khiển tích cực đem lại hiệu quả cao, phù hợp để đạt được các chỉ tiêu ổn định cho trang thiết bị trên phương tiện cơ giới này. Bài báo này trình bày kết quả xây dựng công cụ phân tích dao động trên phương tiện cơ giới gắn gối cách ly dao động, làm cơ sở tính toán, thiết kế gối giảm chấn điện từ nhằm giảm dao động trên phương tiện cơ giới này.

**Từ khóa:** Mô hình động lực học; phương tiện cơ giới đường bộ; gối đỡ giảm rung.

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## 1. Introduction

Currently, a very important issue that needs to be solved is ensuring the smoothness, stability, and vibration reduction of specialized machinery and equipment placed on road motor vehicles. Therefore, the research and application of advanced methods to reduce vibrations for this equipment and machinery placed on motor vehicles is very scientifically and practically significant.

Among the methods of damping oscillations for machine systems, structures, and equipment, the passive oscillation isolation method is most commonly used. The basic principle of the method is to isolate the source of oscillation and structure, which consumes part or all of the energy generated by the agitated source by the element or element system that absorbs and dissipates the energy [1]. This paper presents the results of oscillation analysis [2], [3], placed on a motor vehicle with an oscillation isolation pillow attached, as the basis for calculating the design of electromagnetic damping pillows in order to reduce oscillations on said motor vehicles.

## 2. Methodology of research

### 2.1. Model construction assumptions [3], [4]

To simplify the problem, in Figure 1 below, there are three assumptions: The volume of specialized machinery on the MP car is hypothetically designed to oscillate only vertically, with no axial oscillations in space. The displacement of each axle is considered an independent long displacement of two symmetrical suspension masses, the platform of which is considered to be absolutely rigid. The suspension and tires are subject only to long, linear vertical oscillations. The drag of the mechanics is linearly proportional to the velocity of displacement; the vertical stiffness of the pair of two front and two rear tires is respectively equal.

### 2.2. System of differential equations moving the system [4], [5], [6], [8]

The oscillation reduction study model for machinery placed on road motor vehicles is outlined as a model as shown in Figure 1.

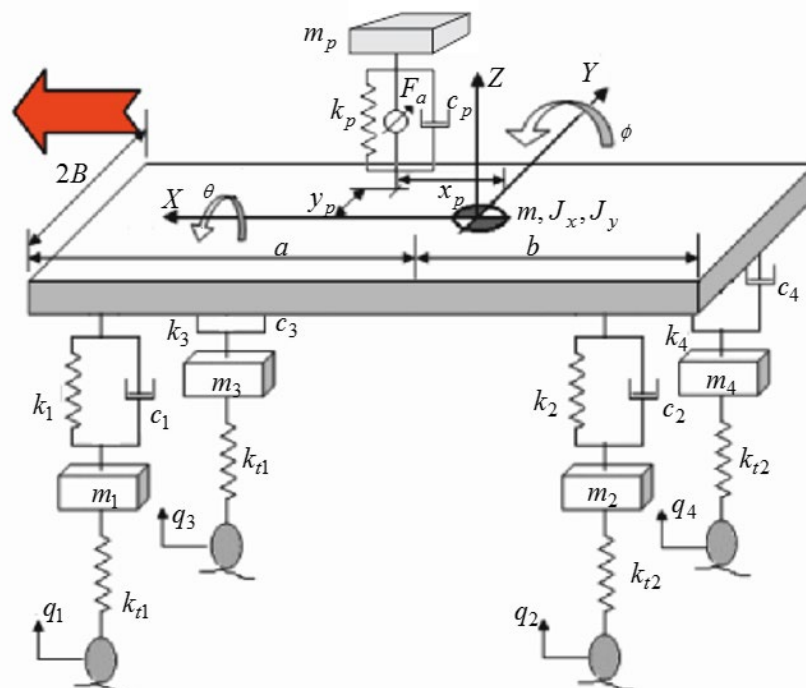


Figure 1. Model of equipment system on road motor vehicles

In which the device has a mass  $m_p$  mounted on the floor of the car through a pillow that reduces electromagnetic oscillations with rigidity  $k_p$ , resistance  $c_p$  and positive control force  $F_a$ , at a position  $OX$  and  $OY$  axes away from the center of mass of the vehicle in the axes  $x_p$  and  $y_p$  respectively. The body has mass  $m$ , the moment of inertia rotating around the  $OX$  axis is  $J_x$ , the moment of inertia rotating around the  $OY$  axis is  $J_y$ ; The front left, front right, left rear and right rear suspensions are modeled in the form of dampers springs with corresponding stiffness and resistance  $k_1, c_1; k_3, c_3; k_2, c_2; k_4, c_4$ . The two front tires of the car have rigidity  $k_{t1}$  and correspondingly the two rear tires of the car have rigidity  $k_{t2}$ . The reaction from a road to two front tires  $q_1, q_3$  and rear tires  $q_2, q_4$  of the car are accordingly.

The kinetic state of the system is defined with 8 degrees of freedom:  $z_p$  - is the movement of the mass of equipment vertically;  $z$  - is the displacement of the body mass vertically,  $\sigma$  - is the rotation of the body mass around the  $OY$  axis,  $\theta$  - is the rotation of the body mass around the  $OX$  axis, and the vertical displacement of the four non-suspended masses is respectively  $z_1, z_2, z_3, z_4$ .

### 3. Results and solutions

#### 3.1. System of differential equations of motion

The system of differential equations of motion of the system consists of 8 equations corresponding to degrees of freedom, including:

$$m_p \ddot{z}_p + k_p (z_p - z - x_p \theta - y_p \varphi) + c_p (\dot{z}_p - \dot{z} - x_p \dot{\theta} - y_p \dot{\varphi}) + F_a = 0 \quad (1)$$

$$\begin{aligned} m \ddot{z} - k_p (z_p - z - x_p \theta - y_p \varphi) - c_p (\dot{z}_p - \dot{z} - x_p \dot{\theta} - y_p \dot{\varphi}) - F_a \\ + k_1 (z - a\theta + B\varphi - z_1) + c_1 (\dot{z} - a\dot{\theta} + B\dot{\varphi} - \dot{z}_1) + k_2 (z + b\theta + B\varphi - z_2) \\ + c_2 (\dot{z} + b\dot{\theta} + B\dot{\varphi} - \dot{z}_2) + k_3 (z - a\theta - B\varphi - z_3) + c_3 (\dot{z} - a\dot{\theta} - B\dot{\varphi} - \dot{z}_3) \\ + k_4 (z + b\theta - B\varphi - z_4) + c_4 (\dot{z} + b\dot{\theta} - B\dot{\varphi} - \dot{z}_4) = 0 \end{aligned} \quad (2)$$

$$\begin{aligned} J_x \ddot{\theta} + k_p (z_p - z - x_p \theta - y_p \varphi) y_p + c_p (\dot{z}_p - \dot{z} - x_p \dot{\theta} - y_p \dot{\varphi}) y_p + F_a y_p \\ + k_1 (z - a\theta + B\varphi - z_1) B + c_1 (\dot{z} - a\dot{\theta} + B\dot{\varphi} - \dot{z}_1) B + k_2 (z + b\theta + B\varphi - z_2) B \\ + c_2 (\dot{z} + b\dot{\theta} + B\dot{\varphi} - \dot{z}_2) B - k_3 (z - a\theta - B\varphi - z_3) B - c_3 (\dot{z} - a\dot{\theta} - B\dot{\varphi} - \dot{z}_3) B \\ - k_4 (z + b\theta - B\varphi - z_4) B - c_4 (\dot{z} + b\dot{\theta} - B\dot{\varphi} - \dot{z}_4) B = 0 \end{aligned} \quad (3)$$

$$\begin{aligned} J_y \ddot{\varphi} + k_p (z_p - z - x_p \theta - y_p \varphi) x_p + c_p (\dot{z}_p - \dot{z} - x_p \dot{\theta} - y_p \dot{\varphi}) x_p + F_a x_p \\ - k_1 (z - a\theta + B\varphi - z_1) a - c_1 (\dot{z} - a\dot{\theta} + B\dot{\varphi} - \dot{z}_1) a + k_2 (z + b\theta + B\varphi - z_2) b \\ + c_2 (\dot{z} + b\dot{\theta} + B\dot{\varphi} - \dot{z}_2) b - k_3 (z - a\theta - B\varphi - z_3) a - c_3 (\dot{z} - a\dot{\theta} - B\dot{\varphi} - \dot{z}_3) a \\ + k_4 (z + b\theta - B\varphi - z_4) b + c_4 (\dot{z} + b\dot{\theta} - B\dot{\varphi} - \dot{z}_4) b = 0 \end{aligned} \quad (4)$$

$$m_1 \ddot{z}_1 - k_1 (z - a\theta + B\varphi - z_1) - c_1 (\dot{z} - a\dot{\theta} + B\dot{\varphi} - \dot{z}_1) + k_{01} z_1 = k_{01} q_1 \quad (5)$$

$$m_2\ddot{z}_2 - k_2(z + b\theta + B\varphi - z_2) - c_2(\dot{z} + b\dot{\theta} + B\dot{\varphi} - \dot{z}_2) + k_{02}z_2 = k_{02}q_2 \quad (6)$$

$$m_3\ddot{z}_3 - k_3(z - a\theta - B\varphi - z_3) - c_3(\dot{z} - a\dot{\theta} - B\dot{\varphi} - \dot{z}_3) + k_{01}z_3 = k_{01}q_3 \quad (7)$$

$$m_4\ddot{z}_4 - k_4(z + b\theta - B\varphi - z_4) - c_4(\dot{z} + b\dot{\theta} - B\dot{\varphi} - \dot{z}_4) + k_{02}z_4 = k_{02}q_4 \quad (8)$$

Use the corresponding state variable vector as follows:

$$\begin{aligned} \mathbf{X} &= [X_1 \ X_2 \ X_3 \ X_3 \ X_5 \ X_6 \ X_7 \ X_8 \ X_9 \ X_{10} \ X_{11} \ X_{12} \ X_{13} \ X_{14} \ X_{15} \ X_{16}]^T \\ &= [z \ \dot{z} \ \varphi \ \dot{\varphi} \ \theta \ \dot{\theta} \ z_1 \ \dot{z}_1 \ z_2 \ \dot{z}_2 \ z_3 \ \dot{z}_3 \ z_4 \ \dot{z}_4 \ z_p \ \dot{z}_p]^T \end{aligned} \quad (9)$$

The system of moving differential equations consists of equations (1) -(8) that can be written as matrix state equations:

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} + \mathbf{B}\mathbf{Q} + \mathbf{G}\mathbf{F} \quad (10)$$

where:

$$\begin{aligned} \mathbf{A} &= [A_1 \ A_2 \ A_3 \ A_4 \ A_5 \ A_6 \ A_7 \ A_8 \ A_9 \ A_{10} \ A_{11} \ A_{12} \ A_{13} \ A_{14} \ A_{15} \ A_{16}]^T \\ A_1 &= [0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]; \\ A_3 &= [0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]; \\ A_5 &= [0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]; \\ A_7 &= [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]; \\ A_9 &= [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]; \\ A_{11} &= [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0]; \\ A_{13} &= [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0]; \\ A_{15} &= [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1]; \\ A_2 &= \begin{bmatrix} \frac{A_{21}}{J_x} & \frac{A_{22}}{J_x} & \frac{A_{24}}{J_x} & \frac{A_{23}}{J_x} & \frac{A_{25}}{J_x} & \frac{A_{26}}{J_x} & \frac{k_1 B}{J_x} & \frac{c_1 B}{J_x} & \frac{k_2 B}{J_x} & \frac{c_2 B}{J_x} & \frac{-k_3 B}{J_x} & \frac{-c_3 B}{J_x} & \frac{-k_4 B}{J_x} & \frac{-c_4 B}{J_x} & \frac{-k_p y_p}{J_x} & \frac{-c_p y_p}{J_x} \end{bmatrix}; \\ A_{21} &= -(k_1 + k_2 + k_3 + k_4)B^2 + k_p y_p^2; \\ A_{22} &= -(c_1 + c_2 + c_3 + c_4)B^2 + c_p y_p^2; \\ A_{23} &= (ak_1 - bk_2 - ak_3 + bk_4)B + k_p y_p x_p; \\ A_{24} &= (ac_1 - bc_2 - ac_3 + bc_4)B + c_p y_p x_p; \\ A_{25} &= -(k_1 + k_2 - k_3 - k_4)B^2 + k_p y_p; \\ A_{26} &= -(c_1 + c_2 - c_3 - c_4)B^2 + c_p y_p; \end{aligned}$$



When vehicles move on the road, agitations from the road surface are the main cause of the system oscillating. Oscillations are measured by sensors, which transmit to the controller, where the controller processes the signal and makes control commands that transmit it to the pillow to reduce electromagnetic oscillations. The controller can use a number of different control laws, such as PID (Programmable Integrated Diagram), LQR (Linear Quadratic Regulator), or intelligent control laws [7].

In order to design these controllers with the goal of reducing oscillations for equipment, it is first necessary to design and control pillows to reduce electromagnetic oscillations and generate positive control forces that satisfy the vibration reduction criteria

### 3.2. Computational model, pillow design to reduce electromagnetic oscillations [8]

Computational model, design of pillows that reduce electromagnetic oscillations are presented on Figure 2.

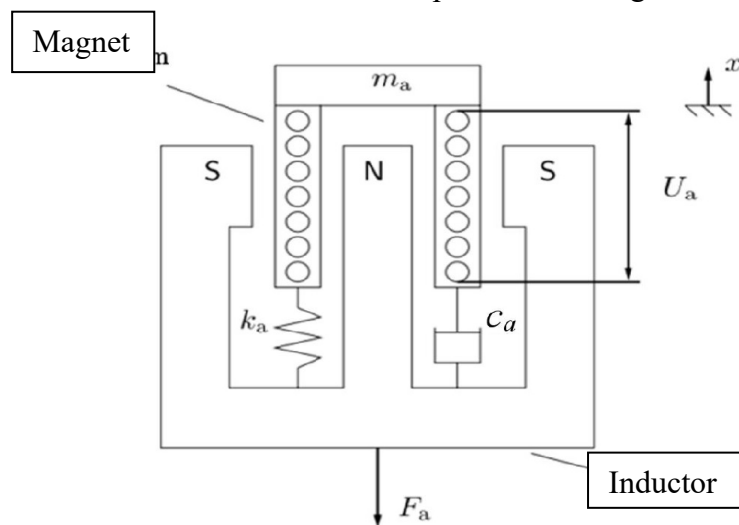


Figure 2. Electromagnetic support pillow models

In this model, force  $F_a$  is the force applied by the isolate pillow due to electromagnetic oscillations,  $m_a$  is the mass of the moving part of the pillow,  $U_a$  is the voltage imposed,  $k_a$  is the rigidity of the pillow,  $c_a$  is the parameter corresponding to the resistance of the pillow. The mass of the induction part is connected to the moving coil in the magnet's magnetic field thanks to the permanent magnet. The moving mass is connected to the floor through rigidity  $k_a$  and resistance  $c_a$ . The amperage in the coil is determined by the equation:

$$Ri + L \frac{di}{dt} = U_a - C_1 \dot{x} \quad (11)$$

where: is impedance;  $R$  is resistance ;  $L$  is the induction of resistance;  $-C_1 \dot{x}$  is dynamic voltage generated by movement. The equation of motion of the displaced mass fraction is:

$$m_a \ddot{x} + c_a \dot{x} + k_a x = F \quad (12)$$

where,  $F$  is the electromagnetic force. This force is proportional to the amperage in the coil in relation to:

$$F = C_2 i \quad (13)$$

Laplace transform the equations from (11) to (13), we have:

$$\left[ m_a s^2 + \left( c_a + \frac{C_1 C_2}{R + sL} \right) s + K \right] X = \frac{C_2}{R + sL} U_a \quad (14)$$

On the other hand, due to the law of inertia  $F_a = -m_a \ddot{x}$ , it should be written in the form of Laplace  $F_a = -m_a s^2 X$ . In combination with equation (4), determine the acting force transfer function of the electromagnetic oscillation isolation pillow  $F$  and the voltage applied:

$$\frac{F_a}{U_a} = \frac{-m_a C_2 s^2}{m_a L s^3 + (m_a R + c_a L) s^2 + (c_a R + k_a L + C_1 C_2) s + k_a R} \quad (15)$$

Thus, the positive force of the pillow reduces electromagnetic oscillations completely determined by the voltage set when the structural parameters of the pillow are known. For the oscillating isolation pillow model for machinery and equipment placed on a motor vehicle, as shown in Figure 1, the displacement of the movable part is:

$$x = z_p - z - x_p \theta - y_p \varphi \quad (16)$$

Or Laplace form

$$X = Z_p - Z - x_p \Theta - y_p \Phi \quad (17)$$

The objective of controlling the oscillating isolation pillow for equipment and machinery placed on the motor vehicle results in adjusting the voltage imposed by  $U_a$  so that its displacement is minimal.

#### 4. Conclusion

Oscillation isolation is one of the two widely used methods to reduce fluctuations and improve the stability of machinery, equipment, buildings, and other equipment. For equipment on motor vehicles subject to agitation from the road surface and powertrain, the reduction of vibrations is especially meaningful to improve stability, improve the safety factor, and open up the possibility of positive oscillation isolation applications. One of the most advanced methods available today for reducing vibrations while helping to improve the working quality of

specialized machinery and equipment (e.g., medical machinery, equipment, radio, television, police equipment, military equipment, etc.). Mechanically, the suspension of motor vehicles is also an oscillating isolator. However, because these systems are designed primarily to ensure oscillation stability for people and cargo under pavement action (background agitation) in a certain frequency zone, most suspensions do not have the appropriate characteristics of specialized equipment. Therefore, it is necessary to add this study to the problem mentioned above.

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